SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2016
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #3

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 hours
- Write using black pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- In Section II answer each of Questions 8 – 13 in a separate booklet. Multiple choice questions (Section I) are to be answered on the answer sheet provided.
- All necessary working should be shown in every question, except multiple choice.

Total Marks – 81

Section I Pages 2 – 4
7 Marks

- Attempt questions 1 – 7.
- Allow about 10 minutes for this section.

Section II Pages 5 – 10
74 Marks

- Attempt questions 8 – 13.
- Allow about 1 hour and fifty minutes for this section.
- Unless otherwise directed give your answers in simplest exact form.

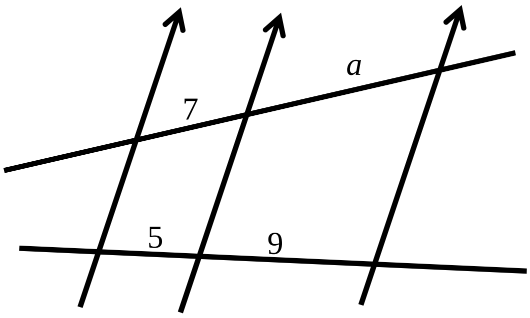
Examiner: *A.M.Gainford*

Section I
Multiple Choice

ANSWER ON THE ANSWER SHEET PROVIDED

In Questions 1 to 7 indicate which of the answers A, B, C, or D is the most correct answer. Write the letter corresponding to the answer on the answer sheet supplied.

Question 1



1

The value of *a* in the diagram above is:

A: 11

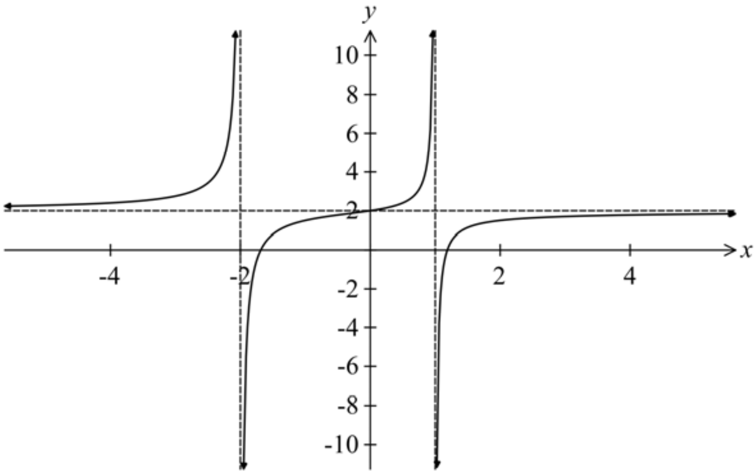
B: 12

C: $12\frac{3}{5}$

D: 13

Question 2

The values of x for which the graph is increasing are: **1**



A: $x < -2$

B: $-2 < x < 1$

C: $x > 1$

D: All of the above.

Question 3

If $\log x = a$, and $\log y = b$ then an expression for $\log\left(\frac{x}{\sqrt{y}}\right)$ is: **1**

A: $\frac{a}{\sqrt{b}}$

B: $\frac{a}{2b}$

C: $a - \frac{b}{2}$

D: $a - 2b$

Question 4

1

The mid-point of the interval joining $P(-5,-3)$ and $Q(2,-1)$ is:

A: $\left(-\frac{7}{2},-2\right)$ **B:** $\left(-\frac{7}{2},-1\right)$

C: $(-7,-4)$ **D:** $\left(-\frac{3}{2},-2\right)$

Question 5

If $e^{x+4} = e^{2x-1}$ then x is equal to: **1**

A: $-\frac{5}{3}$

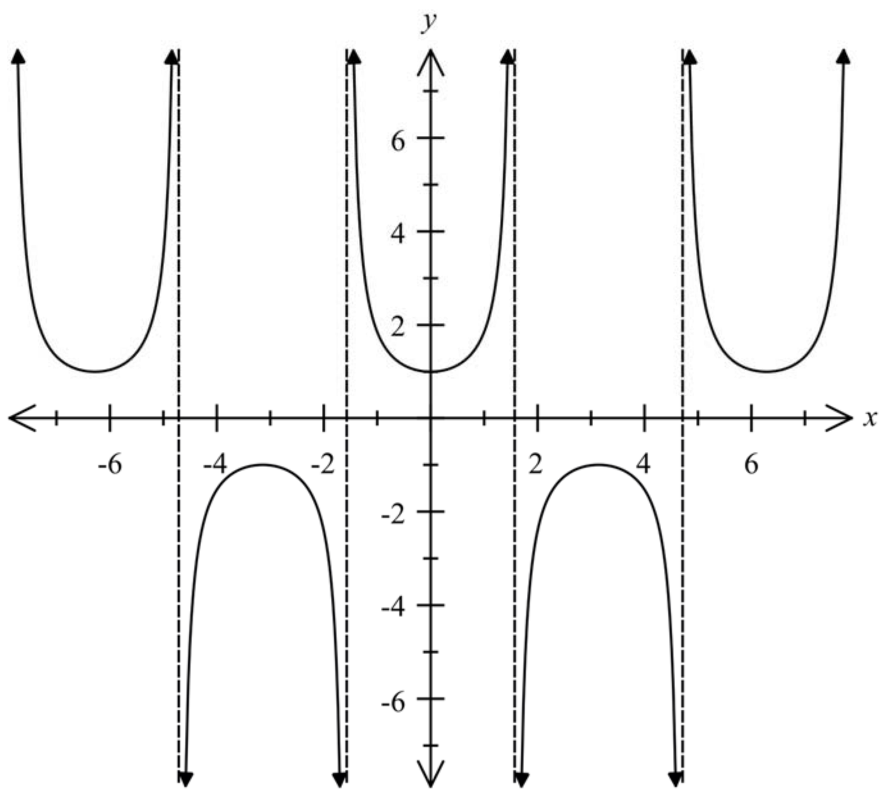
B: 5

C: -5

D: $e^{-\frac{5}{3}}$

Question 6

The graph of $y = f(x)$ is shown below. $f(x)$ could be equal to: **1**



A: $\operatorname{cosec}(-x)$ **B:** $\sec(-x)$

C: $\cos x$ **D:** $\cot x$

Question 7

The area between the curves $y = 2x$ and $y = 6x - x^2$ is given by: **1**

A: $\int_0^4 \left(x^2 + 4x\right) dx$ **B:** $\int_0^4 \left(-x^2 - 4\right) dx$

C: $\int_0^4 \left(x^2 - 4x\right) dx$ **D:** $\int_0^4 \left(4x - x^2\right) dx$

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Section II

Question 8 (12 marks) Start a NEW booklet

(a) Differentiate the following:

(i) $y = \cos 3x$ **1**

(ii) $y = 2\tan^2 x$ **1**

(iii) $y = e^{-2x}$ **1**

(iv) $y = x(\ln x - 1)$ **2**

(v) $y = \dfrac{e^x}{x}$ **2**

(b) Find

(i) $\int \cos(-x)\, dx$ **1**

(ii) $\int \dfrac{e^x}{e^x - 7}\, dx$ **1**

(iii) $\int \sec^2 3x\, dx$ **1**

(c) Evaluate $\int_{\frac{1}{2}}^{1} 2e^{2x-1}\, dx$ **2**

Question 9 (11 marks) Start a NEW booklet

(a) $A(0, 4)$ and $B(-3, 0)$ are points in the number plane. The line through *A* perpendicular to *AB* meets the *x*-axis at *C*.

(i) Sketch the figure on a number plane in your answer booklet.

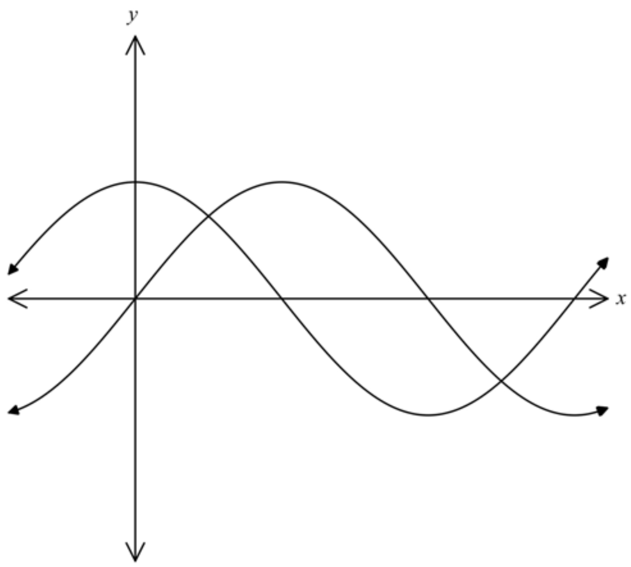
(ii) Show that the equation of the line *AC* is $3x+4y-16=0$. **2**

(iii) Find the coordinates of *C*. **1**

(iv) Find the area of the triangle *ABC*. **1**

(v) The point $D(0, a)$ lies on the *y*-axis below the point *A*. Find the coordinates of *D* if it is 4 units from *AC*. **3**

(b)



The sketch shows the curves $f(x)=\cos x$ and $g(x)=\sin x$.

(i) Find the *x*-values where the curves intersect in $0 \le x \le 2\pi$. **2**

(ii) Find the area enclosed by the curves in the interval between the intersection points on the graph above. **2**

Question 10 (13 Marks) Start a NEW booklet

(a) Given the function $y = \dfrac{1}{x^2 - x - 2}$:

(i) Find the domain of the function. **1**

(ii) Find the stationary point of the function. **1**

(iii) Find $\lim\limits_{x \to \infty} \dfrac{1}{x^2 - x - 2}$. **1**

(iv) Hence, sketch $y = \dfrac{1}{x^2 - x - 2}$. **2**

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \cos 2x \, dx$. **2**

(c) Consider the curve $y = x^3 + x^2 - 2x$:

(i) Find the x-intercepts of $y = x^3 + x^2 - 2x$. **1**

(ii) Find the area bounded by the curve and the x-axis. **1**

(d) (i) Sketch the graph of $y = \sec x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ and shade the area bounded by the curve, the y-axis, the x-axis, and the line $x = \dfrac{\pi}{4}$. **2**

(ii) Find the volume generated by rotating the shaded region about the x-axis. **2**

Question 11 (13 Marks) Start a NEW booklet

(a) Consider the function $y = \dfrac{\log_e x}{x}$.

(i) Find the derivative. **2**

(ii) Hence find the maximum value of $\dfrac{\log_e x}{x}$ and justify your answer. **2**

(b) Given the function $y = x^4 - 2x^3 - 1$:

(i) Find the co-ordinates of the stationary points, and determine their nature. **2**

(ii) Find the co-ordinates of any points of inflexion. **2**

(iii) Sketch the curve, showing the above features. **2**

(c) Use the trapezoidal rule (with five function values) to find an approximation to the following integral: **3**

$$\int_{-1}^{3} 2^x \, dx$$

Question 12 (12 Marks) Start a NEW booklet

(a) A body is moving in a straight line so that its displacement from then origin (x metres) after t seconds is given by:

$$x = \frac{50(t-3)}{e^t}$$

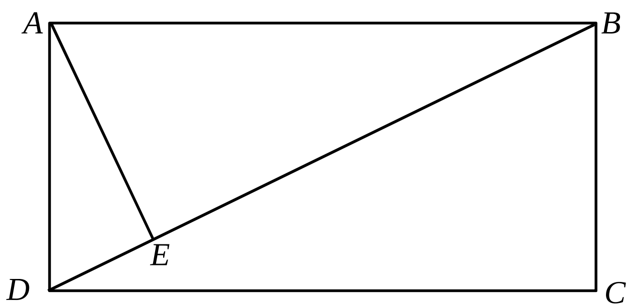
(i) Find its velocity after t seconds. **2**

(ii) Find its initial position. **1**

(iii) Find the greatest positive displacement. **2**

(iv) Find the body's maximum speed. **2**

(b)



The figure $ABCD$ is a rectangle and $AE \perp BD$.
AE = 5 cm and DE = 2 cm.

(i) Copy the diagram to your answer booklet, and prove that triangles AED and BCD are similar. **2**

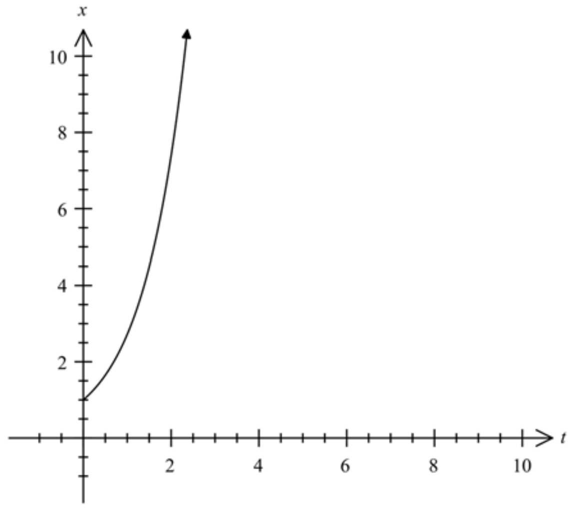
(ii) Hence show that $AD^2 = BD.DE$. **1**

(iii) Find the area of $ABCD$. **2**

Question 13 (13 Marks) Start a NEW booklet

(a) Two particles, P and Q, are moving along a horizontal line. At any time t seconds the position of P is given by $x = e^t$ and the position of Q is given by $x = 1 + 6e^{-t}$.

The diagram below shows the position x (metres) of particle P at any time t.



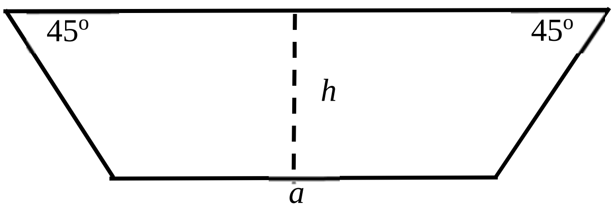
(i) As time increases indefinitely, what position does Q approach? **1**

(ii) Copy the graph to your answer booklet, and sketch the graph of the position of Q on the same diagram. **2**

(iii) Calculate the position where the two particles meet. **2**

(iv) Explain why P and Q will never travel at the same velocity. **2**

(b) A trough of length 1 metre and depth h metres is to be constructed out of stainless steel sheeting. The cross section of the trough, shown below, is an isosceles trapezium. The width of the bottom of the trough is a metres, and the area of the cross section is 60 m^2.



(i) Show that $a = \dfrac{60}{h} - h$. **2**

(ii) Show that the total area of stainless steel is $A = \dfrac{60}{h} - h + 2\sqrt{2}h + 120$. **2**

(iii) Find the depth, to nearest mm, for minimum area of stainless steel. **2**

This is the end of the paper.

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MOORE PARK, SURRY HILLS

2016

Year 12

Assessment Task 3

Mathematics 2U

Suggested Solutions
&
Markers' Comments

QUESTION	Marker
1 – 7	-
8 – 9	**PB**
10 – 11	**JWC**
12 – 13	**JM**

Multiple Choice Answers

1. C
2. D
3. C
4. D
5. B
6. B
7. D

C D C D B B D
1 2 3 4 5 6 7

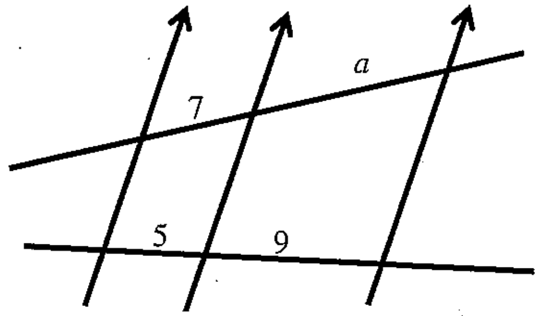
2 unit
4R12 Assess Task

Section I
Multiple Choice

ANSWER ON THE ANSWER SHEET PROVIDED

In Questions 1 to 7 indicate which of the answers A, B, C, or D is the most correct answer. Write the letter corresponding to the answer on the answer sheet supplied.

Question 1



$\frac{7}{a} = \frac{5}{9}$

$5a = 63$

$a = 12\frac{3}{5}$

1

The value of a in the diagram above is:

A: 11

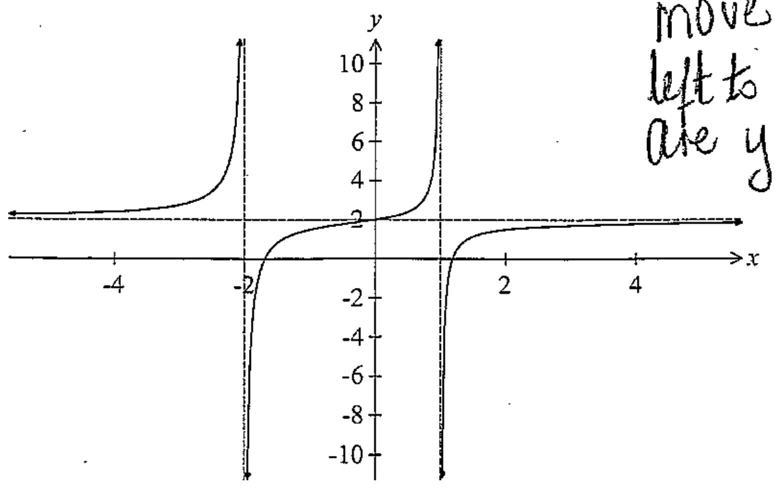
B: 12

C: $12\frac{3}{5}$

D: 13

Question 2

The values of x for which the graph is increasing are: 1



move left to right along x axis. are y values increasing?

A: $x < -2$ **B:** $-2 < x < 1$

C: $x > 1$ **D:** All of the above.

Question 3

If $\log x = a$, and $\log y = b$ then an expression for $\log\left(\frac{x}{\sqrt{y}}\right)$ is: **1**

A: $\frac{a}{\sqrt{b}}$

B: $\frac{a}{2b}$

C: $a - \frac{b}{2}$

D: $a - 2b$

$= \log x - \log y^{\frac{1}{2}}$
$= \log x - \frac{1}{2}\log y$
$= a - \frac{b}{2}$

Question 4

The mid-point of the interval joining $P(-5,-3)$ and $Q(2,-1)$ is: **1**

$M\left(\frac{-5+2}{2}, \frac{-3-1}{2}\right)$

A: $\left(-\frac{7}{2},-2\right)$ B: $\left(-\frac{7}{2},-1\right)$

C: $(-7,-4)$ D: $\left(-\frac{3}{2},-2\right)$

Question 5

If $e^{x+4} = e^{2x-1}$ then x is equal to: **1**

A: $-\frac{5}{3}$

B: 5

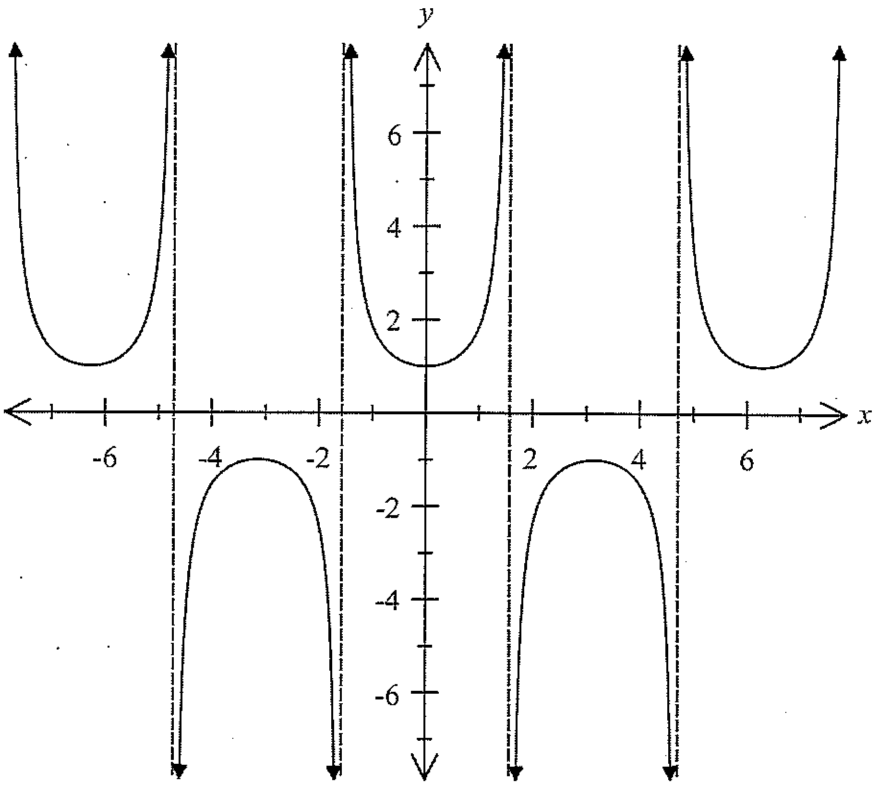
C: -5

D: $e^{-\frac{5}{3}}$

Base is the same 'e'.
equate indices
$x + 4 = 2x - 1$
$5 = x$

Question 6

The graph of $y = f(x)$ is shown below. $f(x)$ could be equal to: **1**



A: $\operatorname{cosec}(-x)$ **B:** $\sec(-x)$ $= \sec x$ (even fn)

~~**C:**~~ $\cos x$ ~~**D:**~~ $\cot x$

Question 7

$-2x$ and $y = 6x - x^2$ is given by: **1**

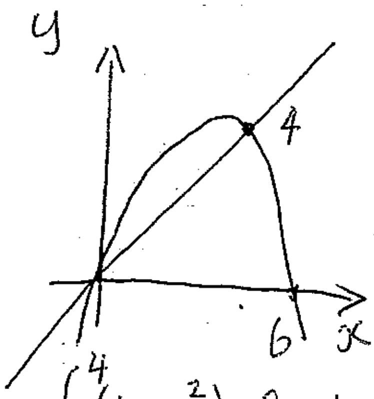
Erratum 2U Multiple Choice

Question 7

The area between the curves $y = 2x$ and $y = 6x - x^2$ is given by:

A: $\int_0^4 (x^2 + 4x)\,dx$ **B:** $\int_0^4 (-x^2 - 4)\,dx$

C: $\int_0^4 (x^2 - 4x)\,dx$ **D:** $\int_0^4 (4x - x^2)\,dx$



$\int_0^4 (6x - x^2) - 2x\,dx$

MATHS QUI

QUESTION 8.

(a) (i) $y = \cos 3x$

$y' = -3\sin 3x$ (1)

(ii) $y = 2\tan^2 x$

$y' = 4\tan x \sec^2 x$. (1)

(iii) $y = e^{-2x}$

$y' = -2e^{-2x}$. (1)

(iv) $y = x(\ln x - 1)$.

$y = x\ln x - x$

$\therefore y' = 1 + \ln x - 1$ (2)

$y' = \ln x$

(v) $y = \frac{e^x}{x}$

$y' = \frac{x e^x - 1 \cdot e^x}{x^2}$ (2)

$\therefore y' = \frac{e^x(x-1)}{x^2}$

COMMENT: Parts (i), (ii) + (iii) were well done. most students received 1 mark.

In part (iv) some misinterpreted the question to read $y = x\ln(x-1)$.. which resulted in the answer $\ln(x-1) + \frac{x}{x-1}$. they were awarded 1 mark.

Part (v) was generally well done.

Q 8(CONTD)

(b) (i) $\int \cos(-x)\,dx = \int \cos x\,dx$ (even function) (1)

$= \underline{\sin x + c}$

(ii) $\int \frac{e^x}{e^x - 7}\,dx = \underline{\ln(e^x - 7) + c}$ (1)

(iii) $\int \sec^2 3x\,dx = \underline{\frac{1}{3}\tan 3x + c}$. (1).

COMMENT: These were all standard integrals and were well answered.

(c) $\int_{\frac{1}{2}}^{1} 2e^{2x-1}\,dx = \left[\frac{e^{2x-1}}{1}\right]_{\frac{1}{2}}^{1}$ (2)

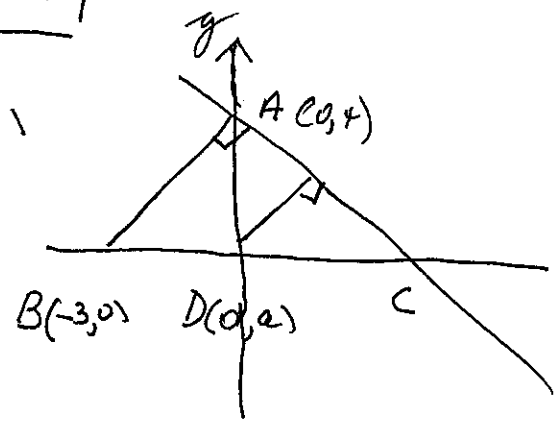
$= e^1 - e^0$

$= \underline{e - 1}$

COMMENT: Well done.

QUESTION 9

(a) (i)



(ii) $m_{AB} = \frac{4-0}{0--3}$

$= \frac{4}{3}$

$\therefore m_{AC} = -\frac{3}{4}$ (2)

$\therefore$ eqn. of AC $y = -\frac{3}{4}x + 4$

$4y = -3x + 16$

ie $\underline{3x + 4y - 16 = 0}$

(iii) C is $\underline{(5\frac{1}{3}, 0)}$ (1)

(iv) Area of $\triangle ABC = \frac{1}{2} \times (5\frac{1}{3} + 3) \times 4.$

$= 2 \times 8\frac{1}{3}$

$= \underline{\frac{50}{3}\ u^2}$ (1)

(v) $\left| \frac{3 \times 0 + 4a - 16}{\sqrt{3^2 + 4^2}} \right| = 4$ (3)

$\frac{|4a - 16|}{5} = 4$

$|4a - 16| = 20$

$4a - 16 = \pm 20$

$\therefore a = 9, -1$

$\therefore \boxed{D \text{ is } (0,-1)}$

since (0,9) is above A.

COMMENT. Parts (ii), (iii) & (iv) were well done.

Part (v) proved more difficult with a penalty of 1 mark for each significant error.

(b) (i) Let $\cos x = \sin x$

$$\therefore \frac{\sin x}{\cos x} = 1 \qquad (2)$$

$$\tan x = 1$$

$$\underline{x = \frac{\pi}{4}, \frac{5\pi}{4}} \quad \text{for } 0 \le x \le 2\pi$$

(ii)

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx = \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= -\left[\cos x + \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= -\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right)$$

$$= -\left(-\frac{4}{\sqrt{2}}\right)$$

$$= \frac{4}{\sqrt{2}} \qquad (2)$$

$$= \frac{4\sqrt{2}}{2}$$

$$= \underline{2\sqrt{2}.}$$

COMMENT.

Some students split the area into various parts which was unnecessary.

One mark deducted for a significant error. $\frac{1}{2}$ mark for each minor error.

QUESTION 10

a) $y = \dfrac{1}{(x-2)(x+1)}$

$D: x \in \mathbb{R} \quad x \neq 2, \; x = -1$

$\frac{1}{2}$ For only $x \neq 2$ and $x \neq -1$

aii)

ii) $y = (x^2 - x - 2)^{-1}$

$y' = -(x^2 - x - 2)^{-2}(2x - 1)$

$= -\dfrac{(1-2x)}{(x^2 - x - 2)^2}$

$y' = 0 \therefore \quad 1 - 2x = 0$

$x = \frac{1}{2}$

x	0	$\frac{1}{2}$	1
y'	$+$	0	$-$

$\nearrow \rightarrow \searrow$

$\therefore$ max $\left(\frac{1}{2}, -\frac{4}{9}\right)$

$\frac{1}{2}$ For correct differentiation or the x-coordinate

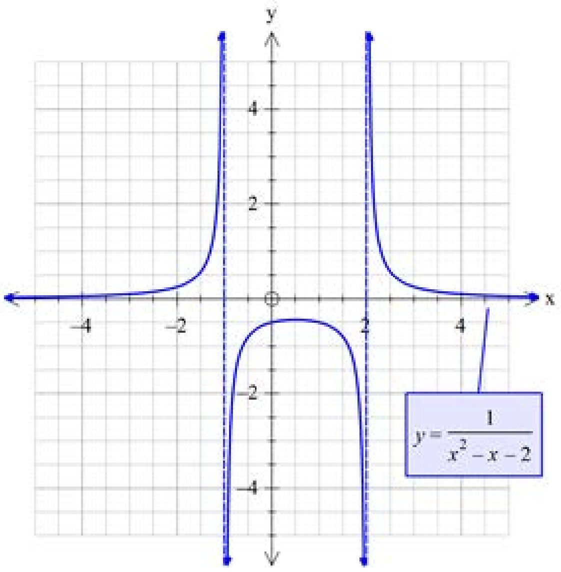
iii)

iii) $\lim\limits_{x \to \infty} \dfrac{1/x^2}{x^2/x^2 - x/x^2 - 2}$

$= \dfrac{0}{1 - 0 - 2}$

$= 0$

aiv)



1 mark for correct graph and $\frac{1}{2}$ for partially correct

$\frac{1}{2}$ clearly marked asymptotes and y-intercept

$\frac{1}{2}$ clearly marked stationary point

bi)

b) $\displaystyle\int_0^{\frac{\pi}{6}} \cos 2x \, dx$

$= \left[\frac{1}{2}\sin 2x\right]_0^{\pi/6}$

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2} \times 0\right)$

$= \dfrac{\sqrt{3}}{4}$

1 mark for correct integration

1 mark answer

ci)

ci) $y = x^3 + x^2 - 2x$

$y = x(x^2 + x - 2)$

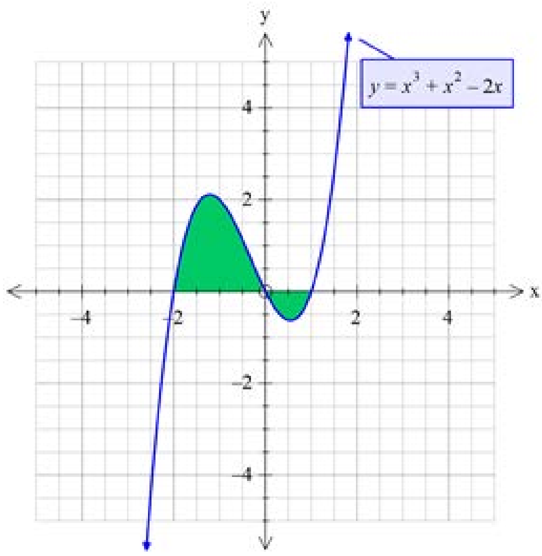
$y = x(x+2)(x-1)$

$y = 0 \therefore x = 0, 1, -2$

Students need to remember x can be zero as well. Students should not use the quadratic formula to find the x-intercepts for this function.

cii)

It will be helpful for student to graph the function in order to understand the area.



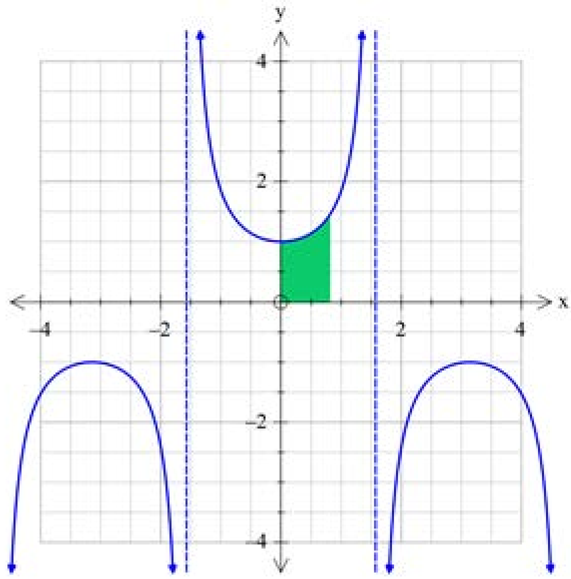
$$A = \int_{-2}^{0} x^3 + x^2 - 2x \, dx + \left| \int_{0}^{1} x^3 + x^2 - 2x \, dx \right|$$

$$= \frac{8}{3} + \left| \frac{5}{12} \right|$$

$$= 3\frac{1}{12} \text{ units}^2$$

$\frac{1}{2}$ mark for either part of the areas $\frac{8}{3}$ or $\frac{5}{12}$

di)



$$V = \pi \int_{0}^{\pi/4} \sec^2 dx$$

$$= \pi \left[\tan x \right]_{0}^{\pi/4}$$

$$= \pi (1 - 0)$$

$$= \pi \text{ units}^3$$

1 mark asymptotes at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

dii)

1 mark for the correct integration

- $\frac{1}{2}$ for not multiplying volume by π

QUESTION 11

a i) $y=\frac{\ln x}{x}$ $\quad$ u v

$y'=\frac{x\left(\frac{1}{x}\right)-\ln x(1)}{x^2}$

$y'=\frac{1-\ln x}{x^2}$

1 mark for correctly applying quotient rule
1 mark for the correct answer

ii)

ii) $y''=\frac{x^2\left(-\frac{1}{x}\right)-(1-\ln x)2x}{x^4}$

$=\frac{-x-2x+2x\ln x}{x^4}$

$y''=\frac{-3x+2x\ln x}{x^4}$

$y'=0$, $1-\ln x=0$

$\therefore x=e$

$f''(e)=\frac{-e}{e^4}<0$

$\therefore$ max value is $1/e$

The maximum is the y-value of the max TP.
1 mark for correctly showing the it is a max TP when $x=e$

bi)

b) $y=x^4-2x^3-1$

i) $y'=4x^3-6x^2$

$y'=0=2x^2(2x-3)$

$x=0$, $x=3/2$

$y''=12x^2-12x$

$f''(3/2)=9>0$

$\therefore$ min $\left(3/2, -2\tfrac{11}{16}\right)$

$\frac{1}{2}$ mark for the correct differentiation
1 mark correctly showing that this is a minimum TP

ii)

ii) $y''=0=12x(x-1)$

$x=0$, $x=1$

x	$-1/2$	0	$1/2$
y''	$+$	0	$-$

$\therefore$ POI $(0,-1)$

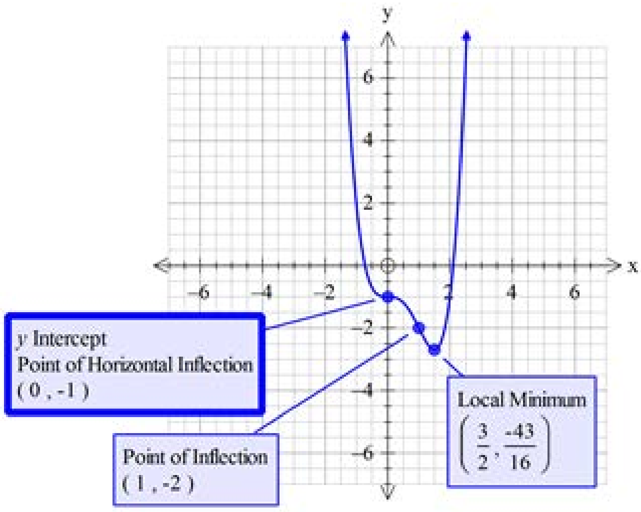
$f'=f''=0$ $\therefore (0,-1)$ is a hort POI.

x	$1/2$	1	$3/2$
y''	$-$	0	$+$

$\therefore$ POI at $(1,-2)$

Students must check concavity for Points of Inflexion. They must state that there is a change of concavity therefore POI. In fact, (0,-1) is a horizontal POI since $y' = y'' = 0$

biii)



c)

x	-1	0	1	2	3
$y=2^x$	$1/2$	1	2	4	8

$\approx \frac{h}{2}(\text{first} + 2\times\text{middles} + \text{last})$

$=\frac{1}{2}\left(\frac{1}{2}+2(1+2+4)+8\right)$

$=\frac{1}{2}\times 22\frac{1}{2}$

$=\frac{45}{4}$

1 mark: the height between each interval is 1 unit
1 mark for 22.5
1 mark for the correct answer

Question 12

(a) $x = \frac{50(t-3)}{e^t}$

(i) $\left(\frac{dx}{dt} =\right) v = \frac{e^t . 50 - 50(t-3) . e^t}{(e^t)^2}$

$= \frac{50e^t - 50te^t + 150e^t}{e^{2t}}$

$= \frac{200e^t - 50te^t}{e^{2t}}$

$= \frac{50e^t(4-t)}{e^{2t}}$

$= \frac{50(4-t)}{e^t}$ [2]

(ii) initial position $(t=0)$:

$x = \frac{50(0-3)}{e^0}$

$= -150$

∴ initially the body is 150 metres to the left of the origin. [1]

(iii) greatest positive displacement $(v=0)$

$\frac{50(4-t)}{e^t} = 0$

$4 - t = 0$

$t = 4$ seconds

when $t = 4$: $x = \frac{50(4-3)}{e^4}$

$= \frac{50}{e^4}$ metres. [2]

$(\approx 0.91578 \text{ m})$.

(iv) maximum speed $\left(\frac{dv}{dt} =\right)$ $a = 0$:

$$a = \frac{e^t \cdot -50 - 50(4-t) \cdot e^t}{(e^t)^2}$$

$$= \frac{-50e^t - 200e^t + 50te^t}{e^{2t}}$$

$$= \frac{-250e^t + 50te^t}{e^{2t}}$$

$$= \frac{50e^t(t-5)}{e^{2t}}$$

$$= \frac{50(t-5)}{e^t}$$

when $a = 0$: $\frac{50(t-5)}{e^t} = 0$

$t - 5 = 0$

$t = 5$ seconds

when $t = 5$: $v = \frac{50(4-5)}{e^5}$

$= \frac{-50}{e^5}$

$\therefore$ speed $(= |v|)$

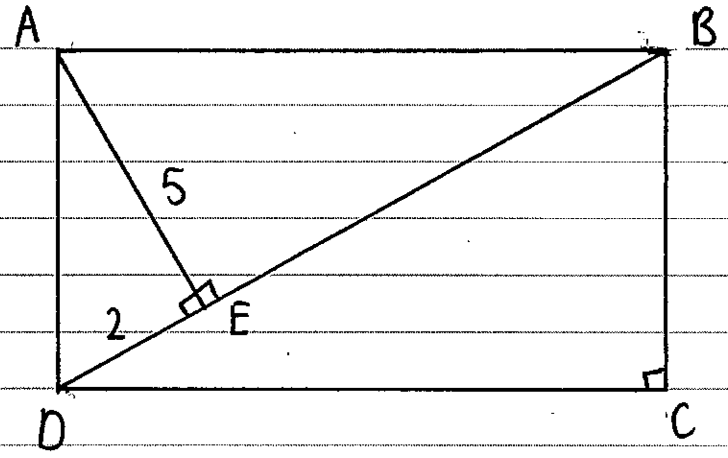
$= \frac{50}{e^5}$ metres/second after 5 seconds. [2]

Comments:

This question was poorly answered. Common mistakes where that students:

- differentiated incorrectly in parts (i) and (iv).
- and the incorrect substitution of values.

(b) (i)



- $AE \perp BD$, $AE = 5$ cm and $DE = 2$ cm.
- $ABCD$ is a rectangle.

In ΔAED and ΔBCD:
- $\angle AED = \angle BCD$ (= 90°, given)
- $\angle CBD = \angle DEA$
  (alternate angles, $AD \parallel BC$)

$\therefore \Delta AED \,|||\, \Delta BCD$ (equiangular) [2]

(ii) $\frac{AD}{BD} = \frac{DE}{AD}$ (ratio of corresponding sides in similar triangles are equal).

$\therefore AD^2 = BD . DE$ [1]

(iii) Area:

$AD^2 = 5^2 + 2^2$

$AD = \sqrt{29}$

$$\frac{AE}{DC} = \frac{DE}{AD}$$

$$\frac{DC}{5} = \frac{\sqrt{29}}{2}$$

$$DC = \frac{5\sqrt{29}}{2}$$

$$\therefore \text{Area}_{(ABCD)} = AD \times DC$$
$$= \sqrt{29} \times \frac{5\sqrt{29}}{2}$$
$$= \frac{145}{2}$$
$$= 72.5 \text{ cm}^2 \qquad [2]$$

Comments:

This question was poorly answered. Common mistakes where that:

- many students did not write the appropriate reasons for proving that the two triangle were similar (in part (i)).
- students used the wrong side to calculate the ratios of the sides to obtain the area of the rectangle.

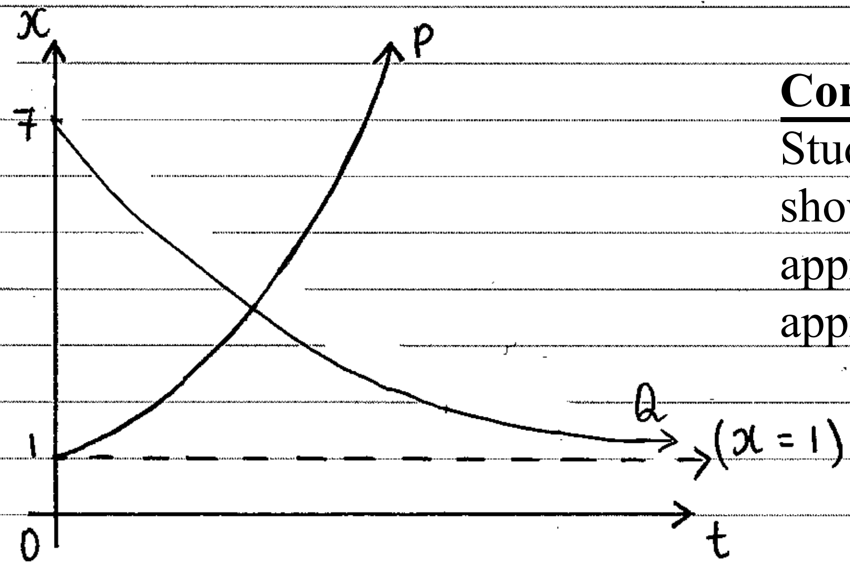
Question 13

(a) P: $x = e^t$
Q: $x = 1 + 6e^{-t}$

(i) As $t \to \infty$, Q: $x = 1 + \frac{6}{e^{\infty}}$ $\left(\frac{6}{e^{\infty}} \to 0\right)$
$= 1 + 0$
$= 1$ [1]

$\therefore x \to 1$m to the right of the origin.

(ii)



[2]

Comments:
Students needed to show that the particle approaches 1 when t approaches infinity.

(iii) Particles meet when P = Q

$$e^t = 1 + 6e^{-t}$$
$$e^{2t} = e^t + 6$$
$$e^{2t} - e^t - 6 = 0$$
$$(e^t - 3)(e^t + 2) = 0$$
$$\therefore t = \ln 3 \quad \text{as} \quad \ln\square > 0$$

when $t = \ln 3$: $x = e^{\ln 3}$
$= 3$

$\therefore$ P and Q meet at $x = 3$ metres. [2]

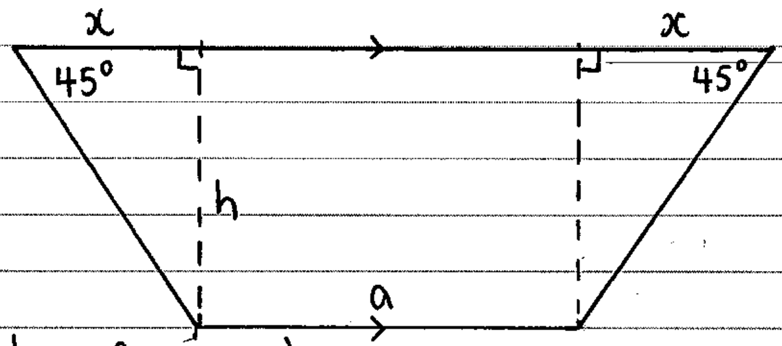
Comments:
Common mistake in this part was made by incorrectly factorising the quadratic.

(iv) Particle P, has a positive gradient $\Rightarrow$ positive velocity (monotonic increasing).

Particle Q, has a negative gradient $\Rightarrow$ negative velocity (monotonic decreasing).

As P and Q are strictly increasing and decreasing, the velocities are never equal. [2]

(b)



(Let $b = 2x + a$)

$\tan 45 = \frac{h}{x}$ $\qquad\qquad$ $A = 60 \text{ m}^2$

$\therefore x = h$

Hence, $b = a + 2h$

(i) $A = \frac{1}{2} h (a + b)$

$60 = \frac{1}{2} h (a + (a + 2h))$

$60 = \frac{h}{2} (2a + 2h)$

$60 = ah + h^2$

$ah = 60 - h^2$

$\therefore a = \frac{60}{h} - h$ [2]

Comments:

Very poorly answered.

(ii) $A = (2 \times \underline{60} + 2 \times \sqrt{2}h + a) \times 1$

$= \dfrac{60}{h} - h + 2\sqrt{2}h + 120$ [2]

(iii) $\dfrac{dA}{dh} = 2\sqrt{2} - 1 - 60h^{-2}$

$= 2\sqrt{2} - 1 - \dfrac{60}{h^2}$

stationary value, when $\dfrac{dA}{dh} = 0$.

$\therefore 2\sqrt{2} - 1 - \dfrac{60}{h^2} = 0$

$\dfrac{60}{h^2} = 2\sqrt{2} - 1$

$h^2 = \dfrac{60}{2\sqrt{2} - 1}$

$h = \pm\sqrt{\dfrac{60}{2\sqrt{2} - 1}}$

($h > 0$, as h is a measure of depth.)

$h \doteqdot 5.728$ m

$\dfrac{d^2A}{dh^2} = 120h^{-3}$

$= \dfrac{120}{h^3}$

when $h \doteqdot 5.728$: $\dfrac{d^2A}{dh^2} = \dfrac{120}{(5.728)^3}$

$= 0.6385$ (>0)

$\therefore$ minimum value when $h = 5728$ mm

$(= 5.728$ m$)$

[2]

Comments:

Students needed to show that the value was indeed a minimum value at h.